Prime Numbers - base patterns...

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A prime number is exactly divisible by two different factors: itself and the number 1

2 3 5 7 Ɛ

The prime numbers {2, 3, 5, 7, 11, 13... (base ten)} have different written forms depending on the number base in which they are written. This has prompted many students to look for patterns for the primes in different bases, but alas! the primes will not conform, and there is no one formula that will produce every prime number.

There *are* patterns to be found, though - in particular, if we look at base six, we find that the prime numbers from 5 upwards are of form 6n+1 or 6n-1; (but this doesn't mean that every number of form 6n±1 is prime).

Here are the first 42 counting numbers arranged in six columns; with the exception of 2 and 3 the primes, printed in ***bold italic***, fall into columns one and five - i.e. belonging to numbers of the form 6n±1.

1	***2***	***3***	4	***5***	6
7	8	9	10	***11***	12
13	14	15	16	***17***	18
19	20	21	22	***23***	24
25	26	27	28	***29***	30
31	32	33	34	35	36
37	38	39	40	***41***	42

If we rewrite our table, with the numbers now written in base six instead of base ten, the pattern becomes much more obvious:

1	***2***	***3***	4	***5***	10
11	12	13	14	***15***	20
21	22	23	24	***25***	30
31	32	33	34	***35***	40
41	42	43	44	***45***	50
51	52	53	54	55	100
101	102	103	104	***105***	110

Other bases of interest are bases four and twelve. Primes in base four are of form 4n±1, and in base twelve 12n±1 and 12n±5.

Here are the first sixteen numbers in base four:

1	***2***	***3***	10
11	12	***13***	20
21	22	***23***	30
31	32	33	100
101	102	***103***	110
111	112	***113***	120
121	122	123	130
131	132	***133***	200

and the first six dozen in base twelve:

1	***2***	***3***	4	***5***	6	***7***	8	9	↊	↋	10
11	12	13	14	***15***	16	***17***	18	19	1↊	***1↋***	20
21	22	23	24	***25***	26	***27***	28	29	2↊	***2↋***	30
31	32	33	34	***35***	36	***37***	38	39	3↊	***3↋***	40
41	42	43	44	***45***	46	47	48	49	4↊	***4↋***	50
51	52	53	54	55	56	***57***	58	59	5↊	***5↋***	60

(Note that we use ↊ for **ten** and ↋ for **eleven** in base twelve).

You have to go to 6n±1 to arrive at the *minimum* set to contain *all* primes (except 2 and 3) whilst leaving out the unwanted odd multiples of 3. The form 4n±1 generates an unnecessarily large quantity of numbers. The point is that the primes are tied to locations about the multiples of six, but do not attain their clearest possible labels until expressed in base twelve, which is the *least* base in which all primes terminate with 1 or a prime digit (5, 7 or ↋) for those greater than 3.

Base Twelve and the Prime Numbers

(by Don Hammond, in Dozenal Journal no. 5)

The dozenal base quickly and easily reveals a fundamental property of prime numbers.

For natural numbers in base twelve greater than 3:
- All numbers terminating with even digits are divisible by 2, and so are not prime.
- All odd numbers terminating with 3 or 9 are divisible by 3, and so are not prime.
- There exist prime numbers of two or more digits which terminate with 1, 5, 7 or Ɛ

Hence, the set of natural numbers terminating with 1, 5, 7 or Ɛ must contain all prime numbers greater than 3, and excludes all odd numbers divisible by 3.
It follows that this set is the *minimum* set to contain *all* primes greater than 3.

Re-arranging the terminal digits as 5, 7 and Ɛ, 1 shows the set to be of the form:

$$(6n \pm 1)$$

Therefore, the minimum set of natural numbers to contain all primes is:

$$\{2, 3, (6n \pm 1)\}\ n \in N$$

This last statement is a factual property of prime numbers and is therefore true regardless of the number-base. it also explains the occurrence of 'twin primes', since the only possible positions for primes greater than 3 are 'each side'of the multiples of 6.

The fact that prime-number positions are *completely* controlled by 6 (itself the product of 2 and 3, and the companion of our dozenal base) is often not realized even by those with an interest in the subject. It is never found in school text-books, and even Hall & Knight do not mention it in their 'Higher Algebra', which is regarded as a standard work.

0 1 2 3 4 5 6 7 8 9 ↊ Ɛ 10